# Superdifferential Cuts for Binary Energies

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## Introduction

**Binary-variable energy minimization**

$E(S) = \sum_{ij} m_{ij}\, s_i s_j + R(S)$

Quadratic   Higher-order


Linear (unary) approximation of higher-order terms:

$$R(S) \simeq \langle h, S\rangle \coloneqq \sum_i h_i s_i$$

Iterative minimization via graph cuts (optimally solved if $m_{ij} \le 0$):

$$S^{t+1} = \operatorname{argmin}\ \tilde{E}(S|S^t) \coloneqq \sum_{ij} m_{ij} s_i s_j + \langle h, S\rangle$$

**Bound optimization approach (our approach)**

If $\tilde{E}(S|S^t) \ge E(S)$ and $\tilde{E}(S^t|S^t) = E(S^t)$, then $E(S^{t+1}) \le E(S^t)$.

- *Submodular Supermodular Procedure* (SSP) [Narasimhan+, UAI 05]
- *Auxiliary Cuts* (AC) [Ayed+, CVPR 13]
- *Parametric Pseudo Bound Cuts* (pPBC) [Tang+, ECCV 14]

**Gradient descent approach (not focused)**

- *Fast Trust Region* (FTR) [Gorelick+, CVPR 13]

**Contribution**

- Generalizes previous bound optimization methods (SSP, AC, etc.)

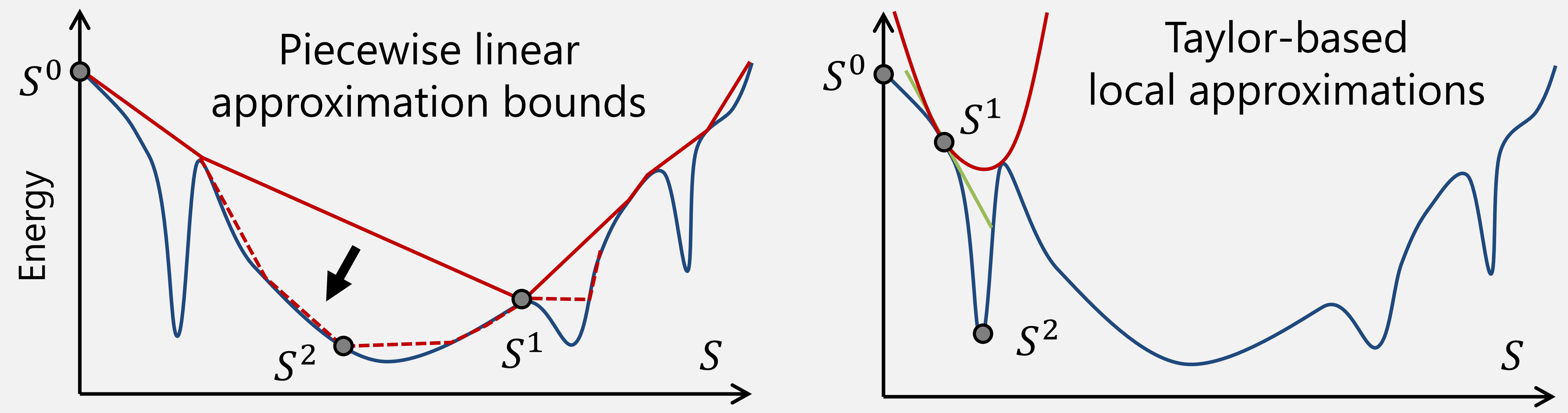
## Scope of Problems

**Type1) Higher-order supermodular**: $R(S) = \sum_z f_z(\langle g_z, S\rangle)$, where $g_z(i): \Omega \to \mathbb{R}^+$ and $f_z(x)$ is convex.
ex) $L_p$-distance of color histograms: $R(S) = \sum_z |n_z^S - hist_z|^p$

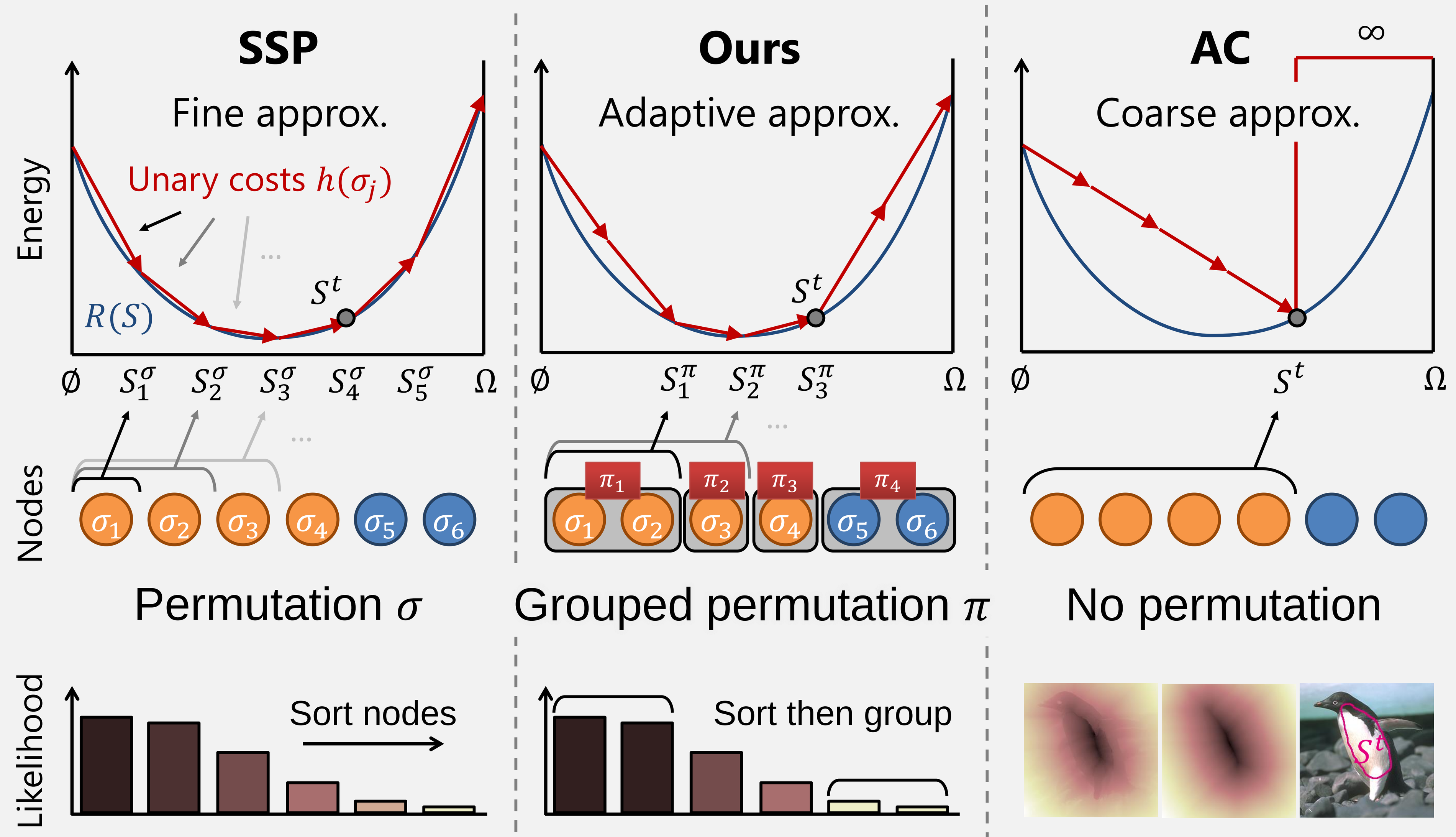
**Type2) Fractional higher-order**: $R(S) = \sum_z f_z(\langle g_z, S\rangle / \langle w_z, S\rangle)$,
ex) KL-divergence and Bhattacharyya coefficient.

**Type3) Quadratic non-submodular**: $m_{ij} > 0$ for some $m_{ij}$ and $R(S) = 0$. QPBO is often used, but it leaves *unlabeled* variables.

## Proposed vs Previous Methods




**Iteratively minimize piecewise-linear approximation bounds that are updated in a coarse-to-fine manner.**

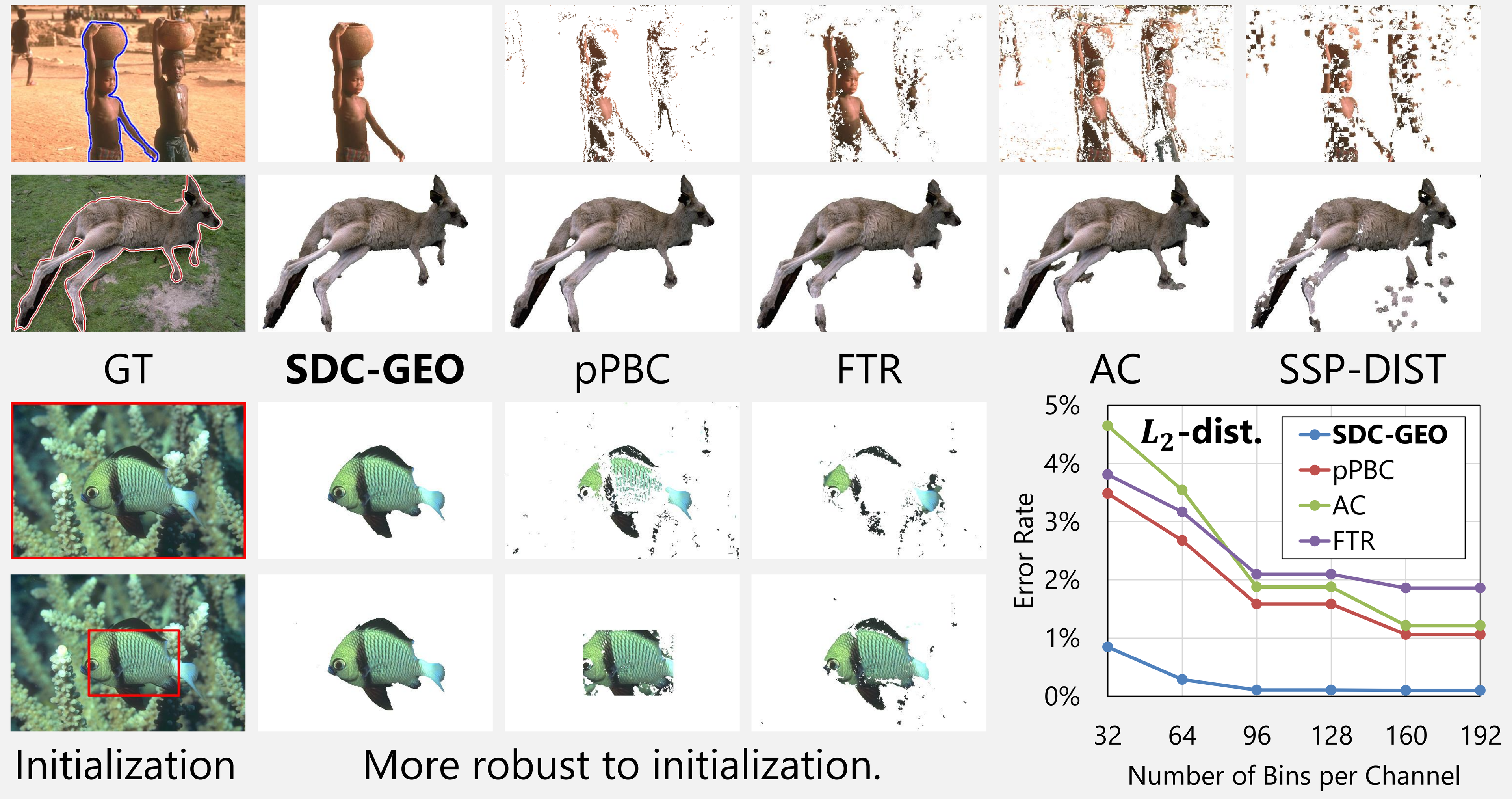



Estimate how likely each variable is "1" by Geodesic/Euclidian distance for $S^t$.

| | SSP | Ours | AC |
| --- | --- | --- | --- |
| Robust. | ☹ Requires very accurate $\sigma$ | ☺ Robust to accuracy of $\sigma$ | ☺ Not require $\sigma$ |
| Accuracy | ☺/☹ Can be best and worst | ☺ Good (Reduced risk) | 😐 Average |

## Experiments

### Type1) Image segmentation via histogram matching




### Type3) Image deconvolution


### Curvature regularization